#### Linear Factor Models

Lecture slides for Chapter 13 of *Deep Learning* www.deeplearningbook.org Ian Goodfellow 2016-09-27

### Linear Factor Models



 $\mathbf{x} = W\mathbf{h} + \mathbf{b} + \mathbf{noise}$ 

Figure 13.1

#### Probabilistic PCA and Factor Analysis

- Linear factor model
- Gaussian prior
- Extends PCA
  - Given an input, yields a distribution over codes, rather than a single code
  - Estimates a probability density function
  - Can generate samples

### Independent Components Analysis

- Factorial but non-Gaussian prior
- Learns components that are closer to statistically independent than the raw features
- Can be used to separate voices of *n* speakers recorded by *n* microphones, or to separate multiple EEG signals
- Many variants, some more probabilistic than others

# Slow Feature Analysis

- Learn features that change gradually over time
- SFA algorithm does so in closed form for a linear model
- Deep SFA by composing many models with fixed feature expansions, like quadratic feature expansion

$$p(\boldsymbol{x} \mid \boldsymbol{h}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{W}\boldsymbol{h} + \boldsymbol{b}, \frac{1}{\beta}\boldsymbol{I}). \qquad (13.12)$$

$$p(h_i) = \text{Laplace}(h_i; 0, \frac{2}{\lambda}) = \frac{\lambda}{4} e^{-\frac{1}{2}\lambda|h_i|}$$
(13.13)

$$\arg\min_{h} \lambda ||h||_{1} + \beta ||x - Wh||_{2}^{2}, \qquad (13.18)$$

## Sparse Coding

| Ø  | Ø.            | ß  | \$            | 3   | G  | 9   | Ŷ   |          | $\mathcal{R}$ | 0 | 5 | 2  | Ŷ  | 0 | 9 | 0 | 5  | ê | 8   |
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| \$ | G             | Ø, | 3             | Ŗ   | 9  | E.  | £   | 8        | 9             | 3 | Ş | 0  | 1  | 3 | 5 | 0 | 10 | 5 | 1   |

Samples

Weights

Figure 13.2

# Manifold Interpretation of PCA



Figure 13.3